Dynamic model of thermal processes in a liquid flowing through a channel

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Summary: The article presents a dynamic thermal process model for an incompressible fluid moving through a channel of arbitrary cross-sectional shape. The study focuses on expressing the non-steady law of conservation of internal energy of the fluid by considering the dependence of parameters on both spatial coordinates and time. The one-dimensional spatial formulation of the problem is simplified by disregarding any changes in flow parameters in the direction perpendicular to the channel axis. The resulting equation represents a non-stationary, non-homogeneous, quasi-linear hyperbolic partial differential equation, which can be solved with the method of characteristics in the case of constant coefficients. The solution consists of two distinct solutions that are glued along the line x/t=v, with isolines shown in the figure provided. The research is relevant for the design of heat exchangers, rocket and aircraft engines, and other heat engineering devices.

Key words: channel flow, dynamic thermal state, non-steady conservation law, quasi-linear hyperbolic PDE, method of characteristics

The focus of this research is to determine the dynamic thermal state of a liquid while it moves through channels, which is an important problem in the design of heat exchangers, rocket and aircraft engines, and other heat engineering devices. There have been numerous studies aimed at solving this problem, as evidenced by previous research (e.g., [123]). However, these studies typically only consider the simplest algebraic relationships between parameters or examine their dependence solely on time or spatial coordinates.

The primary objective of this research is to develop a thermal process model for a liquid that is moving through a channel, which takes into account the dependence of parameters on both spatial coordinates and time.

We are examining the movement of an incompressible fluid in a channel of length l with a cross-sectional shape that is arbitrary (refer to figure 1).

Assuming that the cross-sectional area of the channel is significantly smaller than its length, we can disregard any changes in flow parameters that occur in the direction perpendicular to the channel axis. This allows us to simplify the problem into a one-dimensional spatial formulation. To achieve this, we will establish a coordinate system in which the spatial axis aligns with the channel axis, with the origin located at the point where the liquid enters the channel.
We will then select a control volume, denoted by the symbol $\Omega$, which is formed by planes perpendicular to the channel axis and the walls of the channel. Using this control volume, we will express the non-steady law of conservation of internal energy:

$$\rho S \Delta x (u(t + \Delta t, x) - u(t, x)) = m (u(t, x) - u(t, x + \Delta x)) \Delta t - k \Pi (T(t, x) - T_e) \Delta t \Delta x. \quad (1)$$

Here $t, x$ – temporal and spatial coordinates, $u(t, x) = cT(t, x)$ – specific internal energy of the liquid, $T(t, x)$ – temperature, $\rho, c, m$ – density, specific heat capacity and mass flow rate of a liquid respectively, $T_e$ – ambient temperature, $k$ – heat transfer coefficient from fluid to the environment, $S, \Pi$ – channel cross-sectional area and perimeter.

In the general case, it is evident that all parameters of the liquid and channel may depend explicitly or implicitly on both time and spatial coordinate. The left-hand side of equation (1) represents the change in the internal energy of the liquid in the control volume over a time interval $\Delta t$. Meanwhile, the terms on the right-hand side of equation (1) describe the changes in the internal energy of the liquid within the control volume as a result of the inflow of liquids and heat exchange with the environment through the boundaries of the control volume.

Subsequently, we shall divide both the left-hand and right-hand sides of equation (1) by the product $\Delta t \Delta x$, followed by taking the limit as $\Delta t, \Delta x \rightarrow 0$. This procedure results in an equation that describes the variation of the internal energy of the liquid during its motion within the channel (the independent variables are omitted for simplicity):

$$\rho S \frac{\partial u}{\partial t} + m \frac{\partial u}{\partial x} = -k \Pi (T - T_e). \quad (2)$$

Equation (2) represents a non-stationary, non-homogeneous, quasi-linear hyperbolic partial differential equation. Finally, we will establish the initial and boundary conditions to fully define the problem of fluid flow in a channel:

$$t = 0 \Rightarrow T(0, x) = \varphi(x) \quad (3)$$

$$x = 0 \Rightarrow T(t, 0) = \psi(t) \quad (4)$$
In summary, the problem of determining the thermal state of a fluid flow in a lengthy channel can be fully defined by equation (2) together with the initial and boundary conditions (3, 4).

We will limit our scope to instances of constant coefficients in the problem (2, 3, 4). In this case, equation (2) can be converted into an equation that deals solely with temperature:

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial x} = -b(T - T_e),$$

(5)

Here \(b = \frac{kn}{cs} \) \(v\) – the axial flow velocity.

Solving the problem (5, 3, 4) can be accomplished with ease by using the method of characteristics [4]:

$$T(t, x) = \theta(t - \frac{x}{u}) \left(\psi(t - \frac{x}{u}) - T_e\right) e^{-\frac{bx}{u}} + \theta(\frac{x}{u} - t) \left(\phi(x - ut) - T_e\right) e^{-bt},$$

(6)

where \(\theta(\xi)\) – Heaviside theta-function.

As can be seen from (6), the solution to problem (5, 3, 4) consists of two solutions – the solution to the Cauchy problem (5, 3) and the solution to the boundary problem (5,4). These solutions are glued along the line \(\frac{x}{u} = u\). This is clearly seen in picture 2, where the isolines of the function (6) are shown.

As depicted in expression (6), the solution to problem (5, 3, 4) comprises of two distinct solutions – the solution to the Cauchy problem (5, 3) and the solution to the boundary problem (5,4). These solutions are joined together along the line \(\frac{x}{u} = u\).

The isolines of function (6) can clearly demonstrate this, as shown in figure 2.

Therefore, up until the time \(t_l = \frac{u}{x}\), the temperature distribution throughout the channel can be fully determined by the initial and boundary conditions. For \(t > t_l\), solution (6) is solely determined by the conditions at the entrance of the
channel. As such, if the time interval $t^* \gg t_1$ is being considered in problem (5, 3, 4), the second term in expression (6) can be disregarded.

In conclusion, it can be stated that a model for thermal processes in a liquid flowing through a channel has been developed, and the equations of this model have been solved and analyzed for the case where coefficients remain constant.

References:


